Quantifying Emergent Behaviors in ABM and CA using Conditional Entropy

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Emergence in Complex Systems

In 1984, Wolfram [1] proposed a qualitative four-class scheme for cellular automata (convergent, periodic, chaotic, and complex). However, a quantitative framework for describing emergence remains an open problem. Our aim is to connect these emergent patterns to classical notions of order and chaos in physics.

We propose a method for quantifying the differences between system behaviors by using conditional entropy, also called mean information gain [2]. We show its effectiveness by applying it to a multi-agent biased random walk in a two-dimensional discrete space.

A Biased Random Walk on a 2D Grid

In our toy model, agents move by selecting another visible agent and taking a step towards them. (Fig. 1). Agents behave according to two key mechanisms **vision** and **superposition**.

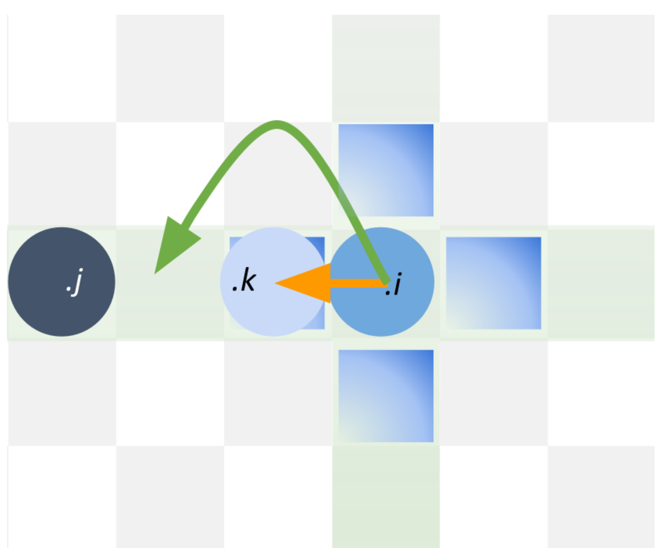

Figure 1:Vision determines which neighbors an agent can interact with: blue patches indicate a Von Neumann neighborhood, while green patches represent an orthogonal neighborhood. Superposition refers to whether multiple agents can occupy the same patch. If agent i moves to agent j, the orange arrow shows movement when superposition is enabled and green when it is not.

By exploring all possible combinations of the parameters, we obtain the complete set of types described in Wolfram's classification (Fig. 2). However, classification is based only on visual inspection.

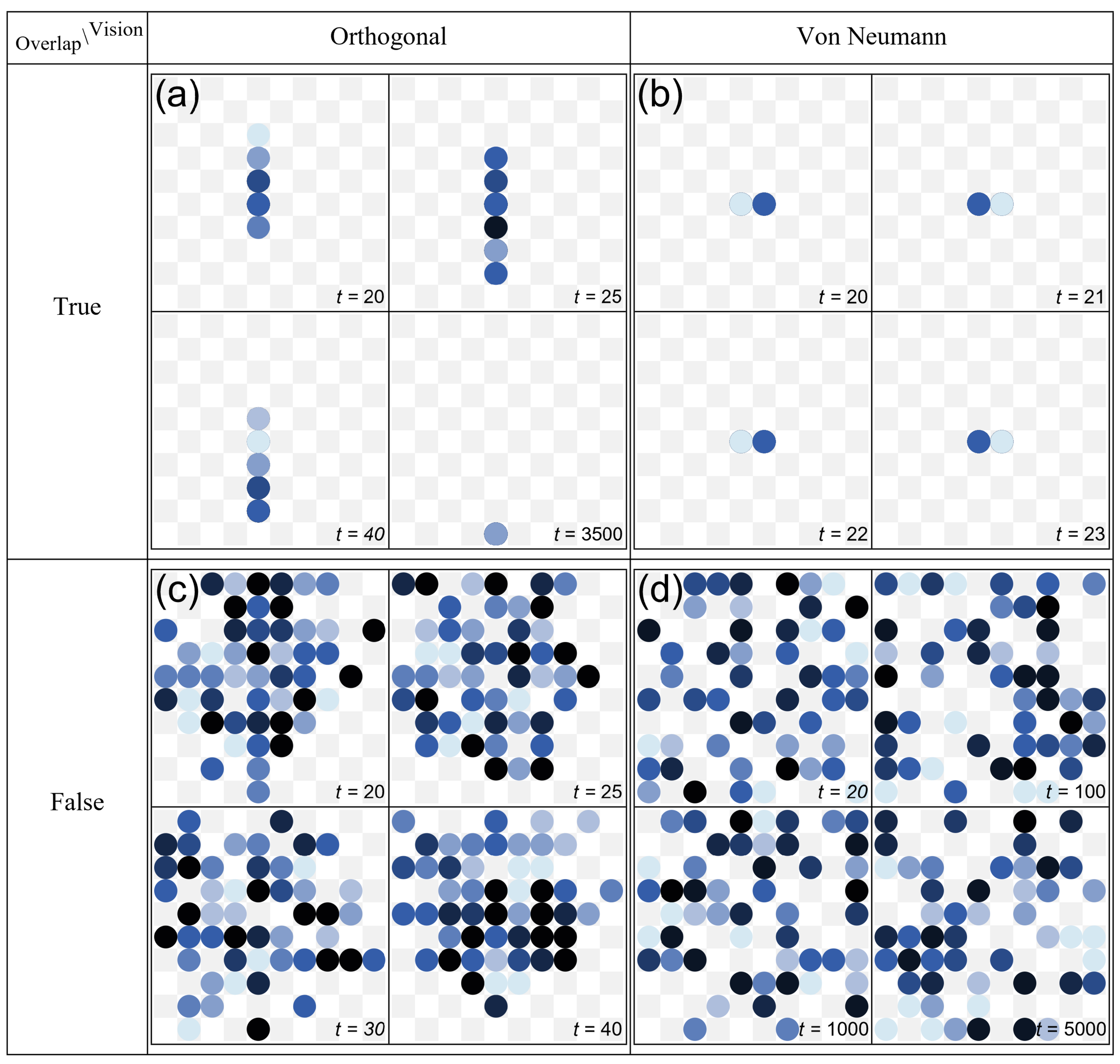

Figure 2:Emergent behaviors: (a) Convergent: settling after oscillation. (b) Periodic: stable clusters. (c) Complex: macro-agent formation. (d) Chaotic: no pattern.

Using Conditional Entropy

To apply the mean information gain to our agent-based model, we map the system onto a cellular automaton: each cell is assigned a binary state 1 if occupied by at least one agent, 0 otherwise. The structural complexity G is defined as the total mean information gain of cells having homogeneous and heterogeneous neighborhood [3]:

$$\overline{G}_{s_r,s_{\Delta r}} = -\sum_{s_r,s_{\Delta r}} P(s_r, s_{\Delta r}) \log_2 P(s_r \mid s_{\Delta r}) \tag{1}$$

Here, s_r is the state of a reference cell, and $s_{\Delta r}$ is the state of a neighbor in one of the four cardinal directions (up, down, left, right).

Results

The mean conditional entropy effectively distinguishes the four type of behaviors and increases with the level of disorder, from convergent to chaotic behavior.

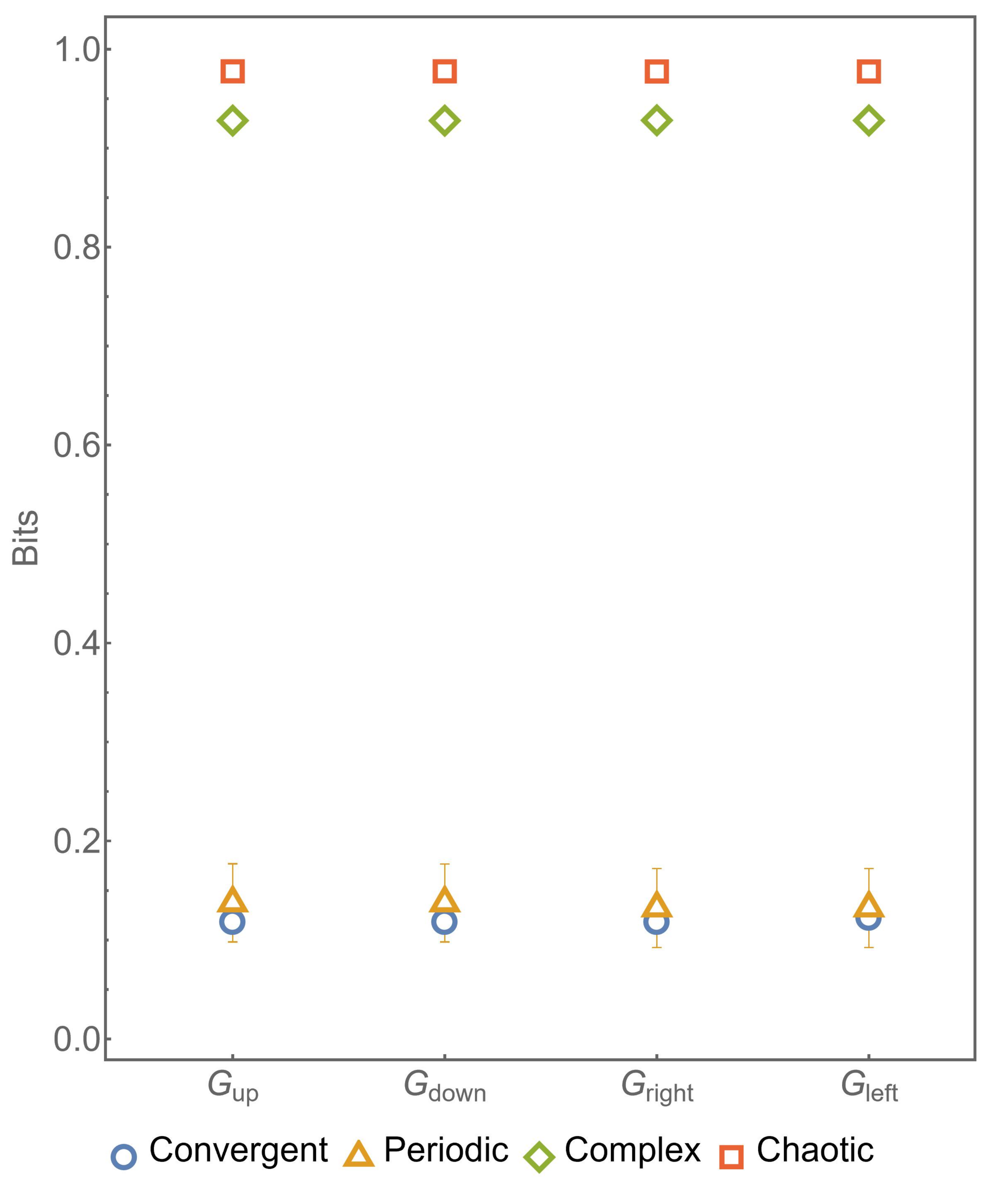

Figure 3:Convergent and periodic systems show low entropy values, with the periodic case maintaining significant standard deviation. Complex and chaotic systems exhibit higher entropy values but different between each other.

Conclusions

- We used a multi-agent biased random walk to reproduce Wolfram's four behavioral classes.
- The system behaves as a macro-agent that performs a collective 2D random walk.
- Mean information gain can be used to distinguish the four regimes and to rank them by spatial disorder over time.

References

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