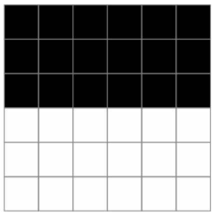
An Informational Model for Cellular Automata Aesthetic Measure

I have solved two of the examples included in the publication, the first one to explain how I am calculating probabilities and the second example is the one I have results reversed.

Example 1



(b)

$$\overline{G}_u = 0.5510$$
$$\overline{G}_d = 0.5510$$
$$\overline{G}_l = 0$$
$$\overline{G}_r = 0$$
$$H = 1$$

First, define the Mean Information Gain for "up" direction where states are S = {w (white) , b (black)} :

$$G_u = -\sum_{i,j_{(x,y+1)}} P(i,\ j_{(x,y+1)})\ \mathrm{Log}_2\ P(i,\ j_{(x,y+1)})$$

$$\begin{aligned} G_u = & -P(w,\ b_{(x,y+1)})\ \mathrm{Log}_2\ P(w \mid b_{(x,y+1)}) \\ & -P(w,\ w_{(x,y+1)})\ \mathrm{Log}_2\ P(w \mid w_{(x,y+1)}) \\ & -P(b,\ w_{(x,y+1)})\ \mathrm{Log}_2\ P(b \mid w_{(x,y+1)}) \\ & -P(b,\ b_{(x,y+1)})\ \mathrm{Log}_2\ P(b \mid b_{(x,y+1)}) \end{aligned}$$

In order to simplify calculations:

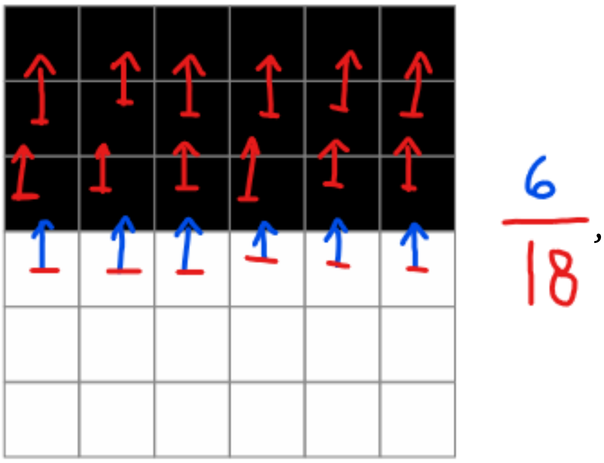
$$P(i,\ j_{(x,y+1)}) = P(j_{(x,y+1)})\ P(i \mid j_{(x,y+1)})$$

Now we only need to calculate $P(j_{(x,y+1)})$ and $P(i \mid j_{(x,y+1)})$ for each case.

Explaning of the first member (i = w, j= b) calculation graphically:

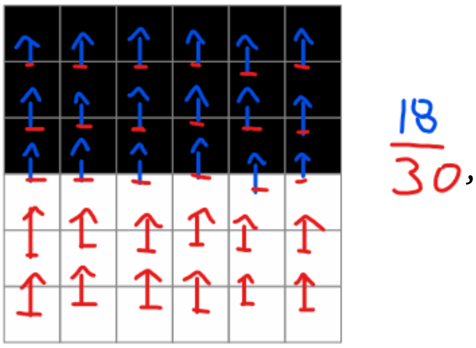
- $P(w,\ b_{(x,y+1)})\ \mathrm{Log}_2\ P(w \mid b_{(x,y+1)})$:

For P (w | $b_{(x,y+1)}$) :



since we have 18 cells with black cells above and only 6 of them are white.

For P ($b_{(x,y+1)}$) :



since we have 18 cells with black cells above and only 30 cells in total with an existant cell above.

Now doing some calculation we got:

In[∘]:=
```
- 18/30 * 6/18 * Log[2, 6/18] // N
```
logaritmo   valor numérico

Out[∘]= 0.316993

Following the same steps, for the following members we can assure that:

- $P(w, w_{(x,y+1)}) \operatorname{Log}_2 P(w \mid w_{(x,y+1)})$:

  P (w | $w_{(x,y+1)}$) = 1 then $\operatorname{Log}_2 P(w \mid w_{(x,y+1)}) = 0 \to 0$

- $P(b, w_{(x,y+1)}) \operatorname{Log}_2 P(b \mid w_{(x,y+1)})$:

  P (b | $w_{(x,y+1)}$) = $0 \to 0$

- $P(b, b_{(x,y+1)}) \operatorname{Log}_2 P(b \mid b_{(x,y+1)})$:

In[∘]:=
```
- 18/30 * 12/18 * Log[2, 12/18] // N
```
logaritmo   valor numérico

Out[∘]= 0.233985

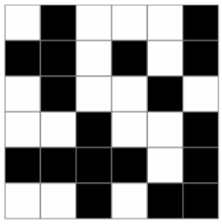
Finally we recover the value $G_u = 0.551$ aprox:

In[ ]:=
```
Gu = -18/30 * 6/18 * Log[2, 6/18] + -18/30 * 12/18 * Log[2, 12/18] // N
```
logaritmo logaritmo va

Out[ ]=
```
0.550978
```

And following the same steps for G_d we will obtain the same result.

Example 2



(d)

$$\overline{G}_u = 0.9839$$
$$\overline{G}_d = 0.9871$$
$$\overline{G}_l = 0.9377$$
$$\overline{G}_r = 0.9473$$
$$H = 1$$

In this example I have recovered all the values but G_R and G_L results are reversed ($G_R = 0.9377$, $G_L = 0.9473$). I will be explaning the procedure as the last example:

First, define the Mean Information Gain for “right” direction:

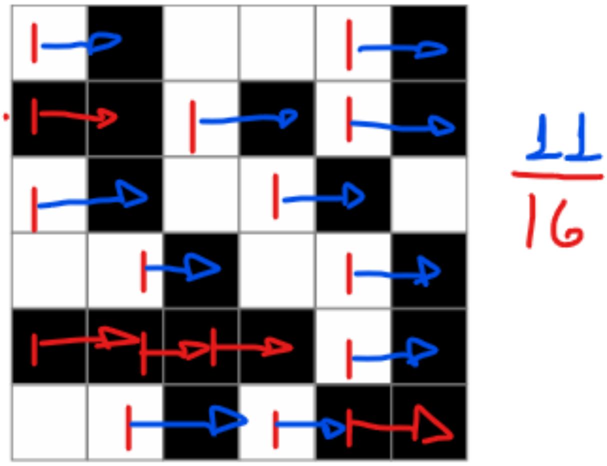
$G_R = -\sum_{i,j_{(x,y+1)}} P(i,\ j_{(x+1,y)}) \operatorname{Log}_2 P(i,\ j_{(x+1,y)}))$

$G_R = \ -P(w,\ b_{(x+1,y)}) \operatorname{Log}_2 P(w \mid b_{(x+1,y)})$
$\quad -P(b,\ b_{(x+1,y)}) \operatorname{Log}_2 P(b \mid b_{(x+1,y)})$
$\quad -P(b,\ w_{(x+1,y)}) \operatorname{Log}_2 P(b \mid w_{(x+1,y)})$
$\quad -P(w,\ w_{(x+1,y)}) \operatorname{Log}_2 P(w \mid w_{(x+1,y)})$

Explanation:

- $P(w,\ b_{(x+1,y)}) \operatorname{Log}_2 P(w \mid b_{(x+1,y)})$:

For $P(w \mid b_{(x+1,y)})$:



, then we now that

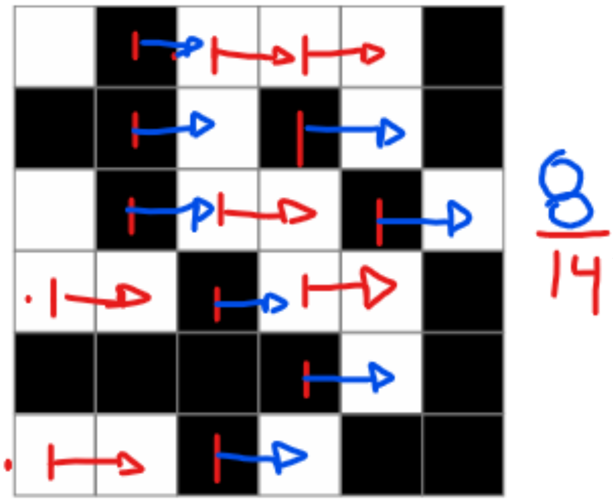
$P(w, b_{(x+1,y)}) = \frac{16}{30} \cdot \frac{11}{16}$

- $P(b, b_{(x+1,y)}) \operatorname{Log}_2 P(b \mid b_{(x+1,y)})$:

  Is the complement case so the result would be $P(b \mid b_{(x+1,y)}) = \frac{5}{16}$ and $P(b, b_{(x+1,y)}) = \frac{16}{30} \cdot \frac{5}{16}$

- $P(b, w_{(x+1,y)}) \operatorname{Log}_2 P(b \mid w_{(x+1,y)})$:

For $P(b \mid w_{(x+1,y)})$:



, then we now that

$P(b, w_{(x+1,y)}) = \frac{14}{30} \cdot \frac{8}{14}$

- $P(w, w_{(x+1,y)}) \operatorname{Log}_2 P(w \mid w_{(x+1,y)})$:

  Is the complement case so the result would be $P(w \mid w_{(x+1,y)}) = \frac{6}{14}$ and $P(w, w_{(x+1,y)}) = \frac{14}{30} \cdot \frac{6}{14}$

Finally we can make the calculation:

In[◦]:=
```
-16/30*11/16*Log[2, 11/16] - 16/30*5/16*Log[2, 5/16] -
  14/30*8/14*Log[2, 8/14] - 14/30*6/14*Log[2, 6/14] // N
```

$$-\frac{16}{30}*\frac{11}{16}*\text{Log}\left[2,\frac{11}{16}\right]-\frac{16}{30}*\frac{5}{16}*\text{Log}\left[2,\frac{5}{16}\right]-\frac{14}{30}*\frac{8}{14}*\text{Log}\left[2,\frac{8}{14}\right]-\frac{14}{30}*\frac{6}{14}*\text{Log}\left[2,\frac{6}{14}\right]\ //\ \text{N}$$

logaritmo logaritmo logaritmo logaritmo valor numérico

Out[◦]= 0.93766

As mentioned this is aprox. G_L values and not G_R. Following the same steps you will get G_L = 0.9473 .